Significant digits

When you do a mathematical calculation and your calculator displays a 16-digit answer, how many of those digits should you write down? In science, the numbers you're working with are usually based on measurements, and the number of digits to keep in your answer depends on how many digits you have in your measurements.

Significant digits are digits that come from the measurement. Sometimes a calculation can result in extra zeroes being added to the end of a large number (like 36,000), or extra zeroes being added to the beginning of a small decimal number (like 0.075). Those zeroes are not significant.

Here's an example. Suppose you measure the width of a TV screen to be 1.2 meters. That measurement is made up of 2 digits, so you have 2 significant digits in that measurement. If you convert the width of the TV to centimeters, you get 120 cm. But only two of those digits, the 1 and the 2, are from the measurement. The zero at the end is not a significant digit.

If you convert the width of the TV screen into kilometers, you get 0.0012 km. That number also has two significant digits, because the leading digits did not come from the measurement, and are not significant.

The general rules for determining how many significant digits there are in a number are:

1) Trailing zeroes at the end of a number that does not have a decimal point are not significant (like those in 36,000).

2) Leading zeroes at the beginning of a decimal that is less than 1 are not significant (like those in 0.075).

3) All other digits are significant, including captive zeroes (like those trapped between the 4's in 4004), trailing zeroes after a decimal point (like in 3.200), and all non-zero digits.

Calculations using significant digits

There are two different rules for how many significant digits to keep in your answer when you do a calculation, depending on whether you are doing addition/subtraction or multiplication/division.

Adding and Subtracting:

Look for the number whose final significant digit ends in the farthest decimal place to the left, and then round your answer to that same decimal place. For example, how many digits should we keep in this calculation?

$$720.0 + 5.4 \times 10^2 - 117 - 0.064 = 1142.936$$

It's easier if we write the addition and subtraction vertically, lining up the decimal points. The last significant digit in each value is in ***bold italics***.

$$\begin{array}{lr} & 720.\boldsymbol{0} \\ + & 5\boldsymbol{4}0 \\ - & 11\boldsymbol{7} \\ - & 0.06\boldsymbol{4} \\ \hline & 11\underline{\boldsymbol{4}}2.936 \end{array}$$

Since 5***4***0 has its last significant digit in the 10's place, your answer should be rounded to the 10's place: 11***4***0

Multiplying and dividing:

When multiplying and dividing, all you do is see which number has the fewest significant digits, and then round your answer to that same number of digits.

In this problem

$$\frac{0.834 \times 52.48}{0.095 \times 2.51} = 183.5534$$

The 0.095 in the denominator has only two significant digits, and that limits your answer to two significant digits: 180

Mixed operations

Things get a little trickier when you have a problem that mixes addition or subtraction with multiplication or division. Look at this monster:

$$\frac{2516.32 \times 1.13 \times 10^{-4}}{6.00} + 8.7 \times 10^{-2} - 6.842 \times 0.0030 = -1.9182093$$

How many of those digits should be reported in the final answer?

Order of operations says to do all the multiplication and division first, and then do the addition and subtraction.

The fraction at the beginning of the equation calculates out to be 0.0473907. Since this fraction is made up of only multiplication and division, all you have to do is count the number of digits in each number to see how many to keep. ***1.13***$\times 10^{-4}$ and ***6.00*** each have three significant digits, and that's what limits the number of significant digits in your answer. ***0.0473***907. But do not round yet! All rounding is done at the end of the calculation. Keep all the digits for now, but remember where the last significant digit falls.

Next do the other multiplication: ***6.842*** × 0.00***30*** = ***2.0***526 Since 0.0030 has only two significant digits, your answer here has only two significant digits. But keep them all for now.

With all the multiplication and division done, we're on to addition and subtraction, so we'll do this just like the first one.

The significant digits in each number are in ***bold italics***. Since we're adding and subtracting here, the final answer has to end in the same decimal place as the least precise of the numbers we're working with. Since 2.0526 is significant only to the tenths place, our final answer has to be rounded to the tenths place.

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  0.0473907
+ 0.087
- 2.0526
- 1.9182093
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Keep all the digits during the calculation, but remember how many of them are significant. Do all your rounding at the end of the calculation. If you rounded -2.0526 to -2.1 in the middle of the calculation, you would get a different (and incorrect) answer for your final result.

A few final comments

Significant digits apply only to measurements, estimates, and rounded values. If there are 24 students in your classroom, that is an exact number with no error, and has an infinite number of significant digits, not just 2. Counting does not limit your significant digits, but estimating does. Fractions such as ½ and ¾ are also exact (unless they are estimates).

Sometimes a measurement can be made to a whole number that ends with a zero. Since the zero was part of the measured value, it is significant. For example, if a giant TV screen was measured to be 180 cm across, the 0 is part of the measurement and is significant, but unless you mark it somehow, people will assume that the number has been rounded to the 10's place.

To label the final 0 in a number as significant, you can either put a decimal point after it with nothing following the decimal point, or else underline the last significant zero:

The TV screen is 180. cm wide. 180 cm wide.

If an elephant was weighed the nearest 100 lbs and came out to be 12,000 lbs, you would report it as 12,000 lbs to indicate that the zero in the hundreds place was part of the measurement and is significant.

Be aware that you can gain or lose significant digits when adding and subtracting:

9.5	2 s.d.	11.8	3 s.d.
+ 2.3	2 s.d.	- 11.0	3 s.d.
11.8	3 s.d.	0.8	1 s.d.